Requirements on the measurements of the number of muons to validate mass composition scenarios

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Abstract

The Ultra High Energy Cosmic Ray composition, and their hadronic interactions at the highest energy, have been a subject of intense research since their discovery. However, the degeneracy that they share when looking to the shower observables is hard to break due to the uncertainties in the absolute energy scale and on the hadronic interaction models, among others with a smaller relative importance. In this note we argue that the first two uncertainties may be surpassed by looking into the shower observables energy evolution instead of their absolute value. For this effect, we have developed a method — the Landscape Plots — to check for transitions due to the mass composition evolution with energy. This method can be used to validate/refute a mass composition scenario that is able to explain a set of shower observables (like $X_{\text{max}}$), by looking for transitions in complementary observables (like the $N_{\mu}$, for instance). As the developed method is able to quantify the significance of the transition, we have used it to derive requirements on the number of muons and energy measurement resolutions. For this, we have used the mass composition scenario suggested by the $X_{\text{max}}$ data. From the analysis of this scenario we conclude that the $\text{RMS}(N_{\mu})$ is very sensitive to the transitions in this scenario while the $\langle N_{\mu} \rangle$ presents a poor sensitivity.

Keywords: Extensive Air Shower, Mass Composition Scenarios, Depth of Shower Maximum, Number of muons at the ground

1. Introduction

The mass composition of Ultra High Energy Cosmic Rays (UHECRs) has been an open problem since their discovery. The direct detection of these particles is not possible and one must rely on observables related to the shower of particles produced by the interaction of these extremely energetic particles with the atmosphere. The description of Extensive Air Showers (EAS) depends both on the nature of the primary cosmic ray and on the characteristics of high energy hadronic interactions. While some hadronic interaction features, like jet production at mid-rapidity, are notably described by perturbative-QCD, in EAS the most relevant multi-particle production is done
in the non-perturbative regime of QCD, and no solution from first principles is possible. To overcome this problem, the hadronic interactions at high energy are described by phenomenological models that are fine tuned through accelerator data and are then extrapolated several orders of magnitude up to the UHECRs. This leads to uncertainties that are not negligible and hamper the primary mass composition determination.

With the new data from LHC, the hadronic interaction models have an opportunity to calibrate their results reducing the uncertainty in the extrapolation to higher energy. However, one should bear in mind that the kinematical regime is different for EAS. In EAS the most relevant region of production is the forward one which can not be easily assessed by the current accelerator experiments. Moreover, at the highest energy, almost two orders of magnitude above LHC, there could be unexpected phenomena in the hadronic sector. Therefore, the creation of tools to break the degeneracy between the primary mass composition and the hadronic interaction models is of utmost importance. In this note, we develop a method to assess possible mass composition scenarios through the analysis of the energy evolution of shower observables.

The note is divided in the following way: in section 2 we discuss how to break the degeneracy between primary mass and hadronic interaction models; in the next section (sec. 3) we present a method to validate/refute mass composition scenario through the analysis of the energy evolution of shower observables; afterwards we apply the method to two distinct mass composition scenarios and discuss them (sec. 4); In section 5 we use the method, previously described, to give some requirements on the muon and energy measurement resolutions; Finally, in section 6 we draw some conclusions and make additional remarks.

2. On the disentanglement of mass composition from hadronic models

Usually the evaluation of the primary mass composition in data is done by finding observables where the dependence of the hadronic interaction models is minimised. The influence of the choice of hadronic interaction models is then treated as a systematic uncertainty. The depth of the shower maximum \( X_{\text{max}} \), and the number of muons at the ground \( N_\mu \), have been proposed as the most sensitive variables both to mass composition and hadronic interaction models. While the \( X_{\text{max}} \) can be directly determined from the measurement of the shower electromagnetic longitudinal profiles, the total number of muons, \( N_\mu \), cannot be accessed directly, and in practice a truncated number of muons, \( n_\mu \), is obtained integrating the muon lateral distribution function (\( \mu \)-LDF) in a finite distance range \( [3] \). In this note we will be using \textsc{Conex} [4], which provides only the total number of muons at ground with an energy above \( E_\mu^{\text{th}} = 1 \text{ GeV} \). However, for the purpose of this note, i.e. to develop an analysis method to validate/refuse specific mass composition scenario through the analysis of the evolution of the shower observables with energy, this approach is satisfactory.

As the experiment becomes more mature, the number of shower observables that is available increases. This allows the combination of different shower observables which may break the degeneracy between mass composition and hadronic interaction models. The main aim of these multivariate analyses is to perform consistency tests to probe our knowledge over the shower development description. Therefore, with these analyses we are naturally testing and even constraining the available space parameter for hadronic interaction models. An example of this is shown
in figure 1. Here, we show the average number of muons at ground as a function of the average shower maximum depth for showers at $E = 10^{19}$ eV. Each point, in a given hadronic interaction models, represents a possible mass composition scenario. Let us evaluate, for instance the results obtained for Sibyll2.1 [5]. The most extreme point at the left represents a scenario where only iron showers exist, while the point at the right is for a scenario where there are only protons. The pure helium and nitrogen scenario are also marked for the case of Sibyll. The remaining green dots are the results for bimodal mass composition scenarios from one pure element to another. Inside of this contour are all the remaining complex mass composition combinations amongst the four species: proton, helium, nitrogen and iron.

The most interesting feature of this plot is that while the problem is degenerate when $\langle N_\mu \rangle$ and $\langle X_{\text{max}} \rangle$ are interpreted alone, here almost all the current hadronic interaction models can be distinguished independently of any mass composition, provided that the experimental resolutions are better than the separation between models. Another, important feature of this kind of plot is that by using the first moments of the observables, the statistical uncertainties can be easily reduced by increasing the number of collected events.

While this plot proves to be very powerful to constrain hadronic interaction models, independently of the primary mass composition, the current uncertainty in the absolute energy scale of $\sim 20\%$, translated as a 20% uncertainty in $\langle N_\mu \rangle$ prediction and about 10 g cm$^{-2}$ in $\langle X_{\text{max}} \rangle$, diminishes significantly its previous discrimination power. In fact, this uncertainty affects almost
all analyses that try to evaluate the likelihood of a hadronic interaction models with the data (or a primary mass composition) through absolute measurements.

Another interesting way of constraining the high energy hadronic interaction models is by trying to isolate pure mass composition samples. In this way, one should be able to assess important hadronic interaction parameters for the shower development, such as, the primary cross-section. However, also here the uncertainty on the energy scale makes the selection of this pure mass samples a difficult task. On the other hand, the purity of the selection should be also affected by the current models uncertainty.

On top of this, our $X_{\text{max}}$ measurements suggest, if one chooses to believe in our current hadronic interaction models, that at the highest energies measured by the Fluorescence Detector, we might have a mass composition of intermediate masses states (helium and nitrogen) [6, 7]. As an exercise, we plot in fig. 2, the number of muons expected at ground for helium and nitrogen, using different hadronic models. Since we want to investigate the capability of selecting pure mass composition samples, we have convoluted the distribution with the current experimental resolution on the determination of the number of muons in an event-by-event basis ($\sigma_{N_\mu} = 0.15$). Clearly, in such scenario, the selection of pure mass shower events would have a poor efficiency. Note that in this plot the uncertainty on the energy scale is not present. This additional uncertainty would decrease this efficiency significantly.

![Figure 2: Average number of muons at ground distributions for showers at $E = 10^{19}$ eV. The results are shown for helium (green lines) and nitrogen (orange lines) induced showers for different hadronic interaction models. The distributions have been convoluted with an experimental uncertainty in the number of muons of 15%.

Therefore, we conclude that in order to learn from our data about mass composition and hadronic interaction models one has to become relatively independent of the absolute scales. In
the next section we have developed a method for this purpose, based on the differential analysis of
the energy evolution of the shower observables, such as $N_\mu$ and $X_{\text{max}}$. Through it we become less
sensitive to the uncertainties caused by the different hadronic interaction descriptions (see sec. 3)
and by the uncertainty on the absolute energy scale. It is quite obvious that, as the energy scale
varies slowly with the shower energy, a differential analysis is one way of getting rid of the impact
of this quantity in the mass composition interpretations.

3. Landscape Plots

In this section we present a method conceived to analyse the shower observables evolution with
the primary particle energy and check for transitions in the mass composition. This method should
be, by construction, nearly independent of the absolute energy scale. But we want to argue that,
by looking to the observables energy evolution\(^1\) we become also rather insensitive to the impact
of the current hadronic interaction models. For this we chose to plot the average number of muons at
ground, for different hadronic interaction models, as a function of the primary mass composition,
$\ln(A)$, for $E = 10^{19}$ eV showers, in fig. 3 (left). In the right plot the same quantities but now $\langle N_\mu \rangle$ is
divided by the average number of muons for proton showers in the respective hadronic interaction
model. From the close inspection of both plots one can easily see that while the absolute value
has a significant dependence on the choice of the model (left plot), the plot in the right shows only
a mild dependence (at most of \~ 10\%). This means that if one look for transitions rather than
for absolute values, we can evaluate a mass composition scenarios almost independently of the
current hadronic interaction models.

To explain the method in detail we will use a simple bi-modal mass composition scenario
with an abrupt transition at $E = 10^{19}$ eV, where we have proton pure composition for energies

\(^1\)which is achieved trough ratios, as it will be shown latter on.
below $E = 10^{19}$ eV and pure iron composite above this same energy. For simplicity, during the
description of the method, we will focus on energy evolution of the average number of muons at
ground, $\langle N_\mu \rangle (E)$. However, as it will be shown latter, this analysis can be applied on several other
quantities such as: intrinsic shower fluctuations of the number of muons at ground, $RMS(N_\mu)$,
and the two first moments of the shower maximum depth distribution, $\langle X_{max} \rangle$ and $RMS(X_{max})$
(see Appendix A for the other variables in this particular mass composition scenario).

In the left plot of figure 4 we show the energy evolution of such mass composition scenario
(in black) for the average number of muons. The results for protons and irons, simulated with
QGSJET-II, are in red and blue, respectively, and these will be used through the note as benchmark.
As one clearly sees, the behaviour of $\log(\langle N_\mu \rangle)$ is similar to the one observed in the elongation
rate, $\langle X_{max} \rangle$.

Although the absolute value is sensitive to the mass of the cosmic ray, it is also influenced by
the absolute scale determination and by the shower interactions description (inclusive low energy
hadronic interactions where there is still some uncertainties). For that reason, we will use the
derivative of $\langle N_\mu \rangle$, becoming in this way independent of the absolute value while being sensitive
to mass composition transitions, or to significant changes hadronic interaction current description.

Therefore, it is useful to use the following quantity,

$$\beta = \frac{\log(\langle N_{\mu 2} \rangle)}{\log(\langle N_{\mu 1} \rangle)} \frac{\langle \mu \rangle}{\langle \mu \rangle}$$

(1)

where $\langle N_{\mu i} \rangle$ is the average number of muons at energy $E_i$, and by definition $E_2 > E_1$. Notice
that for single mass composition scenarios, i.e. if no transition occurs, $\beta$ has the same value in
all the energy region. Moreover, it is worth noticing that equation (1) is composed by ratios that
should diminish considerably the impact of experimental systematic uncertainties.

![Figure 4](image-url)

Figure 4: Proton to iron abrupt transition scenario: (left) Average number of muons as a function of the shower
energy; in black is the corresponding data to this mass composition scenario, in blue is iron and in red is proton
induced showers. (right) Landscape $\beta$ plot (see details in text).
Having a variable sensitive to the slope of $\langle N_\nu \rangle$, let us now plot $\beta$ for the proton to iron abrupt transition scenario. For this we build a 2-D plot were we put $E_2$ in the $x$-axis and $E_1$ in the $y$-axis\(^2\). In the $z$-axis we will put the evaluation of $\beta$, given by eq. 1 for every pair $(E_2, E_1)$. The result for this scenario is displayed in the right plot of figure 4. For $E_1 = E_2$ the equation (1) has an indetermination, and thus it is put to zero. Furthermore, it is easy to see that the value of beta in $(E_2, E_1)$ is the same one as in $(E_1, E_2)$. Since the information is symmetrically replicated we choose to put $\beta = 0$ for $E_1 > E_2$.

The slope of the average number of muons at ground is calculated between $E_1$ and $E_2$. So, for $E_1 \approx E_2$ the slope, $\beta$, is being evaluated locally. Moreover, in the regions that fulfil $\log(E_2) = \log(E_1) + \Delta \log(E)$, $\beta$ was calculated for the same energy intervals $\Delta \log(E)$.

In this particular mass composition scenario one can see (in fig. 4(right)) that $\beta$ has a constant value for all energies except at $\log(E) = 19$, the energy where the fast transition occurs. Since $\beta$ is a measurement of the slope, then the closer $E_1$ is from $E_2$ the bigger is the value of $\beta$. Ideally, all information of this plot is in its diagonal (i.e. when $E_2 \approx E_1$), so this should be the best region to look for transitions. However, due to experimental resolutions and statistical fluctuations, the transitions may be better determined at larger $\Delta \log(E)$.

We will designate this kind of plot onwards as Landscape Plot, as its reading gives information about the structure of $\langle N_\nu \rangle$ for different probing scales.

![Figure 5](image-url)  
Figure 5: (left) Number of shower events as a function of primary energy in bins of $\log(E) = 0.1$. (right) Error of $\beta$ Landscape map (see details in text).

So, the close inspection of the Landscape Plots allow us to identify regions where there were transitions associated to mass composition changes, independently of the absolute value of the measured quantity. However, the significance of the transition cannot be assessed directly from this plot. Thus, it is useful to transform the Landscape Plot using the following relation,

\(^2\)this choice is arbitrary.
\[ D_{\beta,i,j} \equiv \frac{\beta_{i,j} - \beta_{i-1,j-1}}{\sqrt{\sigma^2_{i,j} + \sigma^2_{i-1,j-1}}}, \tag{2} \]

with the \textit{i-index} and the \textit{j-index} associated to the energy bins of \( E_2 \) and \( E_1 \), respectively. In this transformation the transitions significance are evaluated for different \( \Delta \log(E) \) intervals, while being weighted by the error on \( \beta \) determination, \( \sigma \). It is important to note that the construction of this plot implies the comparison of \( \beta_1 \) and \( \beta_2 \) at a fixed energy intervals, \( \Delta \log(E) \). Thus, the evaluation of a transition through \( D_{\beta,i,j} \), only makes sense following the diagonals of fig. 4 (right). In this way \( E_1 \) is no more a free number but is bounded to \( E_2 \) through \( \log(E_1) = \log(E_2) - \Delta \log(E) \).

The determination of \( \sigma_{i,j} \) is achieved through the error propagation of equation (1), and it has an explicit dependence on the number of events used to obtain \( \langle N_\mu \rangle \), \( n \), on the experimental resolution in the determination of the energy and \( \langle N_\mu \rangle \), \( \sigma_E \) and \( \sigma_{N_\mu} \), on the energy range, \( E_2 / E_1 \), and on \( \beta \) itself. Thus, for each \( \beta \) there should be an associated error given by,

\[ \varepsilon_\beta = \frac{1}{\log \left( \frac{E_2}{E_1} \right)} \sqrt{\left( \sigma^\text{rel}_{N_\mu} \right)^2 + \left( \sigma^\text{rel}_{N_\mu} \right)^2 + \beta^2 \left( \left( \sigma^\text{rel}_E \right)^2 + \left( \sigma^\text{rel}_E \right)^2 \right)} \]  \( \tag{3} \)

where \( \sigma^\text{rel}_E \) and \( \sigma^\text{rel}_{N_\mu} \) are the relative uncertainty on the energy and number of muons determination, respectively and are given by,

\[ \sigma^\text{rel}_{N_\mu} = \frac{\sigma_{N_\mu}}{N_\mu \sqrt{n}} \] \( \tag{4} \)

\[ \sigma^\text{rel}_E = \frac{\sigma_E}{E \sqrt{n}} \] \( \tag{5} \)

To be more realistic we use for the number of events as a function of the energy, \( n(E) \), the integral of a power law with a spectral index of \(-2.7\). The normalisation of this function is done in a way such that for \( \log(E) \in [18.95; 19.05] \) there are 100 shower events\(^3\).

Using the \( n(E) \) displayed in figure 5 (left) and taking \( \sigma^\text{rel}_E \) and \( \sigma^\text{rel}_{N_\mu} \) to be around 15\%, one ends up with the 2-D plot shown in fig. 5 (right) for the absolute error in \( \beta \) determination. As expected, the error is bigger when \( E_1 \approx E_2 \), as it is more sensitive to local fluctuations. Nevertheless, one can observe in the \( \varepsilon_\beta \) plotl structures that reveal the complexity of the problem when looking for an adequate energy interval.

We can now build the corresponding \textit{Landscape Plot} in terms of transition significance, \( D_\beta \), using equation 2. The results for \( D_\beta \) for this particular mass composition scenario is shown in figure 6 (left). For simplicity, we present also some profiles of the 2-D plot in fig. 6 (right). The profiles are obtained following the dashed diagonal lines in fig. 6 (left), so that the energy interval in which \( \beta \) is determined remains constant. The line marked as (1) has a smaller \( \Delta \log(E) \) (more sensitive to local structures), while the line (4) is more sensitive to larger energy scale changes. A

\(^3\) roughly the equivalent to one year of the full SD array operation.
close inspection of figure 6 (right) reveals that a strong transition occurs around $E = 10^{19}$ eV, as expected, with a significance greater than 10σ, for the claimed uncertainties and number of events. It is worth noticing that although $\beta$ is maximum when making the determination along the line (1), i.e. locally, the transition significance is greater when determining $\beta$ with a large $\Delta \log(E)$, in this case for the line (4).

Thus, through the Landscape Plots one can check for mass composition transitions for all energy range scales and choose the one with the highest significance. For that we just need to select, for each mass composition scenario, the maximum in module of the $D_\beta$ plot.

This kind of analysis, shown above for the average number of muons at ground, can easily be extended for the fluctuations of the number of muons at ground, $RMS(N_\mu)$, simply changing $\langle N_\mu \rangle$ in equation (1) by $RMS(N_\mu)$ and noticing that the statistical error of the $RMS$ goes with $(\sqrt{2(n-1)})^{-1}$ instead of the usual $(\sqrt{n})^{-1}$, that is used for average value. Moreover, the analysis can also be extended to other shower observables such as the shower maximum depth, $X_{max}$. In this case, the slope of the $\langle X_{max} \rangle$ is given by,

$$E.R. = \frac{\langle X_{max2} \rangle - \langle X_{max1} \rangle}{\log \left( \frac{E_2}{E_1} \right)}$$  \hspace{1cm} (6)

which is the normal definition of the elongation rate of a shower. For the sake of the shortness of the note, all the Landscape Plots of $RMS(N_\mu)$, $\langle X_{max} \rangle$ and $RMS(X_{max})$, for this mass composition scenario, are shown in Appendix A. Since in this note we are mostly interested in checking whether we can observe transitions due to mass composition evolution in several shower observables, we will use as reference plot the one shown in figure 7. Here we plot the maximum significance achieved in some energy scale as a function of the primary particle energy. The
two first moments of the two shower observables, $X_{\text{max}}$ and $N_\mu$, studied in this note, are shown simultaneously for comparison.

![Graph](image)

Figure 7: Proton to iron abrupt transition scenario: Maximum transition significance as a function of the primary particle energy for: $\langle X_{\text{max}} \rangle$, $\text{RMS}(X_{\text{max}})$, $\langle N_\mu \rangle$ and $\text{RMS}(N_\mu)$.

All the explored quantities, $\langle X_{\text{max}} \rangle$, $\text{RMS}(X_{\text{max}})$, $\langle N_\mu \rangle$ and $\text{RMS}(N_\mu)$ present a coherent transition signal at $E \approx 10^{19}$ eV, with significances greater than 10\sigma. Moreover, one can see that in this mass composition scenario the $\text{RMS}(N_\mu)$ would be the best variable to observe a very fast transition from pure proton to pure iron.

Finally, it is of the utmost importance to note that this method does not aim to find transitions in data but to predict them under the assumption of a given mass composition scenario and for a specific hadronic interaction model\(^4\). Therefore, it is valid to adopt the best energy interval scale, at which the transition will be best observed, by choosing the scale that maximises the significance of this transition. Naturally, the direct comparison of the predictions and the data would imply a careful statistical analyses to correctly assess the significance of the results. Nevertheless, the current proposed method offers the possibility of creating prescriptions for our data and to give an assessment of the necessary experimental resolution requirements to validate/refute a mass composition scenario.

\(^4\)although the importance on the choice of the hadronic interaction models is considerably diminish through the arguments presented in fig. 3.
4. Mass Composition Scenarios

The method presented in the previous section was created to identify transitions on the evolution of the moments of shower observables, for a given mass composition scenario. Its main aim is to validate/reject mass composition scenarios independently of absolute scales, namely of the energy absolute scale, and of the hadronic interaction models (absolute total number of muons and absolute depth of the shower maximum). This can be easily done in the following way: if a mass composition scenario is able to explain for instance the energy evolution of $\langle X_{\text{max}} \rangle$ and $\text{RMS}(X_{\text{max}})$ then using the Landscape Plots one could be capable of predicting the expected transitions in the $\langle N_\mu \rangle$ and $\text{RMS}(N_\mu)$, and perhaps, even more importantly, predict the strength of such transitions. Hence, we have analysed several mass composition scenarios with the Landscape plots method. From these we have chosen two mass composition scenarios to present in this note:

- Proton to Helium linear transition
- Mass composition scenario suggested when interpreting the $X_{\text{max}}$ data with Sibyll2.1

Within the several scenarios tested these two were the ones chosen essentially by two reasons: firstly, both scenarios show mass evolution with small variations in the $\ln(A)$, which means that the variables proportional to $\ln(A)$, like $\langle X_{\text{max}} \rangle$ and $\langle N_\mu \rangle$, have small transitions that are naturally harder to detect; secondly, the evolution of $\langle X_{\text{max}} \rangle$ and $\text{RMS}(X_{\text{max}})$ as a function of the primary energy, resembles the ones found in the Auger data\(^5\). In this way, these two scenario provide a good test both for the method capability and to assess the requirements on experimental resolutions.

4.1. Proton to Helium Transition

Our proton to helium mass composition was conceived in a way such that from $E = 10^{18}$ eV to $E = 10^{18.5}$ eV the showers are composed uniquely by protons; at this energy the fraction of helium in data start to rise linearly and reach 100% at $E = 10^{19.5}$ eV; since this is a bimodal composition scenario the fraction of protons will decrease linearly from $E = 10^{18.5}$ reaching 0% in one order of energy magnitude. The results for such scenario in $\langle X_{\text{max}} \rangle$, $\text{RMS}(X_{\text{max}})$, $\langle N_\mu \rangle$ and $\text{RMS}(N_\mu)$ are shown in fig. 8 in black. The proton and iron induced showers lines are shown in red and blue, respectively. The Auger $X_{\text{max}}$ data points are also shown along with its systematic (dashed lines) and statistical (full lines) errors. An interesting feature of this mass composition scenario is that if one ignores the absolute values, it has the same trend as the Auger $X_{\text{max}}$ data. While in the average $X_{\text{max}}$ this is a feature common to many composition scenarios, for the $\text{RMS}(X_{\text{max}})$ it is not. In fact, it is not very easy to find scenarios where the $\text{RMS}$ is monotonously decreasing. This is easy to understand from the formula for the total $\text{RMS}$ of two distributions, see eq. (7). From this expression one can see that in order to continuously decrease the average values of the two distributions must be very close of each other, so that the third term of equation (7) may have a negligible contribution. This, in principle, implies that the dispersion of masses, $\sigma_{\ln(A)}$, has to be small.

\(^5\)in the second mass composition scenario this feature appears naturally by construction.
\[ RMS^2(X_{\text{max}})(\alpha) = (1 - \alpha) RMS^2(X_{\text{max}})_1 + \alpha RMS^2(X_{\text{max}})_2 + \alpha (1 - \alpha) (\langle X_{\text{max}} \rangle_1 - \langle X_{\text{max}} \rangle_2)^2, \tag{7} \]

So, the \( RMS(X_{\text{max}}) \) is currently providing an important constraint to mass composition scenarios. Looking now to the expected number of muons (bottom plots of fig. 8) one can see that the transition on the \( \langle N_\mu \rangle \) is relatively small, while the changes in \( RMS(N_\mu) \) appear to be more noticeable with the same monotonous decreasing behaviour that is found in \( RMS(X_{\text{max}}) \).

Let us now try to assess the significance of the observed transitions using the Landscape Plots. For this we will use again a realistic \( n(E) \) with a reasonable number of events (100 events at \( E = 10^{19} \) eV) and we put \( \sigma_{N_\mu} = \sigma_E = 0.15^6 \). This is shown in figure 9 where we plot the maximum transition significance as a function of the shower energy (all the intermediate plots relevant to the construction of this one are shown in Appendix B).

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6this value was chosen as a reference value for representing the current Auger analysis uncertainties.
From the plot in fig. 9 it is possible to see that no signal (transition) is observed for energies smaller than $10^{18.5}$, as expected. Around $E = 10^{18.6}$ eV the average value of both $X_{\text{max}}$ and $N_\mu$ present the highest signal in all energy range. As see in fig. 8 (bottom-left) the $\langle N_\mu \rangle$ has a small power for the detection of such transition. In fact, with the current experimental uncertainties one should detect a signal no greater than $2\sigma$. From all four variables studied, the $\text{RMS}(N_\mu)$ is the only that is clearly sensitive to this mass composition scenario. Through the analyses of $\text{RMS}(N_\mu)$ energy evolution, one should be able to detect a transition with a significance of up to $4.5\sigma$. It is important to note that the strength of the signal depends directly of the experimental conditions. In other words should the experimental uncertainties improve or should the statistical data sample increase, the higher the significance will be, as it will be discussed in section 5.

Another interesting feature that can be readily seen in fig. 9 is that the transitions in the average value occur near the place where the mass composition transition started. For the $\text{RMS}$ it happens latter and the signal due to this transition often visible over large energy ranges. This is something to have in mind when trying to extract information about the mass composition. However, this interpretation in terms of mass composition, is currently out of the scope of this note. Here, the main purpose is, as stated before, to test mass composition scenarios and use the data to validate or refute them. Thus, the use of Landscape Plots to interpret the data in terms of mass composition should be a subject for a future note.
4.2. $X_{\text{max}}$ data suggested scenario

Another mass composition scenario, in our opinion, crucial to understand the requirements of Auger next steps, is the one provided by the $X_{\text{max}}$ data when interpreted with the current hadronic interaction models. In [6] this fitting procedure was done for several hadronic interaction models and different mass composition assumptions. One of the best fits is the one obtained when using Sibyll2.1 and a four-composition model of: proton, helium, nitrogen and iron. For these reason these will be the mass composition scenario that will be tested. The fraction of each element as a function of the primary particle energy is shown in figure 10. In this plot we show also the $\langle \ln(A) \rangle$ for each energy (in orange) and, as stated before, the evolution of this quantity is rather small. Although the absolute value changes with the chosen hadronic interaction model, the evolution of this quantity, $\langle \ln(A) \rangle$, is essentially the same for all models, i.e. increases in about 1 unit [6, 7].

The results for this particular scenario in $\langle X_{\text{max}} \rangle$, $RMS (X_{\text{max}})$, $\langle N_\mu \rangle$ and $RMS (N_\mu)$ are shown in fig. 11 in black. The experimental points of the $X_{\text{max}}$ moments are shown in the top plots along their respective error bars. The comparison between the fit (line in black) and the data points (black dots) show a good agreement within the systematic uncertainties. The predictions of this mass composition scenario in the two first moments of the number of muons at ground, $\langle N_\mu \rangle$ and $RMS (N_\mu)$, are shown in the bottom plots. As in the previous scenario, the transitions in $\langle N_\mu \rangle$ are smaller than for the $RMS (N_\mu)$. This qualitative statement derived from the inspection of the plots in fig. 11 is confirmed quantitatively by the transition maximum significance plot shown in fig. 12.

Figure 10: Mass composition scenario obtained when interpreting the Auger $X_{\text{max}}$. The curves show the fraction of proton (in black), helium (in red), nitrogen (in green) and iron (in blue) as a function of the primary particle energy. In orange it is displayed the equivalent $\langle \ln(A) \rangle$ also as a function of energy and its scale in on the right (in orange).
Figure 11: $X_{\text{max}}$ mass composition scenario: (top-left) $\langle X_{\text{max}} \rangle$, (top-right) $\text{RMS}(X_{\text{max}})$, (bottom-left) $\langle N_\mu \rangle$, (bottom-right) $\text{RMS}(N_\mu)$ as a function of primary particle energy. The black line represents the data for this mass composition scenario, while the red and blue lines are for proton and iron induced showers, respectively.

The maximum significance graph is now a complex plot, with several transitions regions. However, one should note that this is a very complex mass composition scenario with many transitions, as shown in fig. 10. Curiously, the $\text{RMS}(N_\mu)$ is the most sensitive to the transitions in this scenario while its average value is the less sensitive of the four studied evolutions. $X_{\text{max}}$ moments present transitions with significances that are, in the best cases, between $4 - 6 \sigma$.

The most significant transition is given by the $\text{RMS}(N_\mu)$ at $E \approx 10^{19}$ eV. This transition can be easily seen and understood by looking at the Landscape Plot for the slope of the $\text{RMS}(N_\mu)$, shown in fig. C.34 in Appendix C. This strong transition is clearly associated to the drop of the $\text{RMS}(N_\mu)$ around $E \sim 10^{19}$ eV, and is related with the extinction of proton primaries (see fig. 10).

The average of $X_{\text{max}}$ and $N_\mu$ are directly proportional to $\ln(A)$. This means that, in a scenario where several transitions occur but $\ln(A)$ is conserved, the average values of these two shower observables will have a small sensitivity for these transitions. In such scenarios, the energy evolution of the intrinsic fluctuations of the shower observables will be ones providing the best information.
Figure 12: $X_{max}$ mass composition scenario: Maximum significance as a function of the primary particle energy for: $\langle X_{max} \rangle$, RMS($X_{max}$), $\langle N_\mu \rangle$ and RMS($N_\mu$).

Finally, one of the reasons for the discrepancy between $X_{max}$ and $N_\mu$, in term of transition significance, arises from the fact that the experimental uncertainties in $X_{max}$ are significantly smaller than the ones we have in the number of muons determination. In the next section (sec. 5) we shall explore the impact of these uncertainties on the assessment of mass composition transitions. With this study one should be able to derive some requirements on the experimental uncertainties for muon measurements.

5. Requirements for muon measurements

As seen previously, the scenario drawn from the $X_{max}$ data, when interpreted with Sibyll2.1, is one of the most difficult scenarios to check for mass composition transitions. Moreover, $\langle N_\mu \rangle$ has the worst performance for identifying transitions, within the current experimental uncertainties. Thus, as this is one of the worst case scenarios, and on top, the one suggested by Auger $X_{max}$ data when believing in the current hadronic interaction models, we will use this mass composition scenario and the information that one can attain through the energy evolution of $\langle N_\mu \rangle$ to study the impact of the experimental uncertainties on the determination of transitions.

In section 3, we have develop a method to assess mass composition scenarios, being this one, by construction, rather independent of the absolute measurement value. One of the major advan-

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7we have used the average uncertainties on these observables, currently claimed by Auger.
tages is to be quite insensitive to the absolute energy scale, which makes, with the current 20%, any claim on mass composition or on hadronic interaction models validity quite difficult.

The evaluation of a given mass composition scenario depends basically on three main parameters when studying the \( \langle N_{\nu} \rangle \) evolution with energy: the resolution in the determination of the number of muons at ground event-by-event, \( \sigma_{N_{\nu}} \), the resolution of the energy measurement, \( \sigma_{E} \), and the number of shower events collected, \( n \) (as we are using in this analysis the moments of the shower observables distributions).

As stated before, the number of events as a function of the shower energy, shown in figure 5 (left), was chosen to roughly mimic the number of events collected in a year of a full SD array operation\(^8\). Let us then use this \( n(E) \) to evaluate the impact of \( \sigma_{N_{\nu}} \) and \( \sigma_{E} \) on the capability of observing a transition, given the mass composition scenario described in 4.2, with \( \langle N_{\nu} \rangle \). For this, we will take the maximum value of the maximum transition significance, shown in fig. 12, and plot it as a function of \( \sigma_{N_{\nu}} \) and \( \sigma_{E} \). The resulting 2-D plot is shown in figure 13 and one can clearly see that the significance of the transition is affected both by the uncertainties on the energy and the number of muons measurement, as one should expect.

![Figure 13](image)

**Figure 13**: Maximum transition significance as a function of the resolution on the number of muons and the energy resolution, for the \( \langle N_{\nu} \rangle \) in mass composition scenario described in sec. 4.2. These results were obtained assuming \( n(E) \) as shown in fig. 5 (left).

The 2, 3, and 4 \( \sigma \) contour lines are also shown in the plot as dashed white lines and are accordingly labeled. The contours are not symmetric, indicating that the impact of the energy

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\(^8\)notice that these are all the events collected by the SD, without any analysis quality cut
uncertainty is not the same as for $\sigma_{N_p}$. In fact, for a given $\sigma$-contour, one has that $\sigma_E - \sigma_{N_p} > 0$, and the difference between them increases as the transition significance decreases.

With $\sigma_{N_p} = \sigma_E = 0.15$ we would observe at best a transition of a little more than $2\sigma$, as shown before. To reach the $3\sigma$ contour line we need to push the $\sigma_{N_p}$ down to around 10%, while the $\sigma_E$ needs only to be improved, in this particular scenario, by a few percent.

Figure 14: Maximum transition significance as a function of the number of shower events in $\log(E) \in [18.95, 19.05]$ for the $\langle N_{\mu} \rangle$. Three cases are displayed: $\sigma_{N_p} = \sigma_E = 0.15$ (black/full line), $\sigma_{N_p} = \sigma_E = 0.1$ (red/dashed line) and $\sigma_{N_p} = 0.25$, $\sigma_E = 0.15$ (green/dotted line). (see details in text)

The main features of the $RMS(N_{\mu})$ and $\langle N_{\mu} \rangle$ are essentially the same, but they differ signifi-

18
cantly in the transition significance values. The 9σ contour lines of the $RMS(N_\mu)$ plot lies nearly in the region where the 3σ line is for the $\langle N_\mu \rangle$. This shows, undoubtedly, the capabilities of the intrinsic shower fluctuations of the number of muons at ground, $RMS(N_\mu)$, in testing the mass composition scenario when interpreted with SIBYLL. Of course, with this we are also testing the performance of the high energy hadronic interaction model. Let us suppose that the $RMS(N_\mu)$ refutes the preferable mass composition scenario of a hadronic interaction model. This would naturally imply that the hadronic interaction model has problems explaining our data and therefore its capability of describing hadronic interactions should be, at least, incomplete.

The capability of the $RMS(N_\mu)$ in such scenario is so remarkable that we want to show the previous plot with a quarter of the number of shower events shown before ($n = 25$). The results are shown in figure 15 (right) and, from this, it becomes evident that we would still be able to see a transition, assuming $\sigma_{N_\mu} = \sigma_E = 0.15$, of more than 3σ. Of course, this analysis is completely dependent of our understanding of the detector and our capability to deconvolute the intrinsic shower fluctuations from the experimental ones. But, the inspection of figure 15, assures us that this effort is certainly worth it.

![Figure 15: Maximum transition significance as a function of the resolution on the number of muons and the energy resolution, for the $RMS(N_\mu)$ in mass composition scenario described in sec. 4.2. These results were obtained assuming $n(E)$ as shown in fig. 5, scale by a factor such that in the energy bin of $10^{19}$ eV there are 100 events (left plot) or 25 events (right plot).](image)

Therefore, we have plotted also, in figure 16, the maximum transition significance as a function of the number of shower events in $log(E) \in [18.95, 19.05]$ for the $RMS(N_\mu)$, as it was done in fig. 14 for the $\langle N_\mu \rangle$. It is very interesting to note that even assuming $\sigma_{N_\mu} = 0.25$, $\sigma_E = 0.15$ one can reach a transition significance of 3σ with roughly 100 shower events. The 5σ would be reached by doubling the number of shower events. While the number of events needed is relatively small, one should note that the number of events in an experimental analysis, would have to pass additional quality cuts.
Figure 16: Maximum transition significance as a function of the number of shower events in $\log(E) \in [18.95, 19.05]$ for the $RMS(N_\mu)$. Three cases are displayed: $\sigma_{N_\mu} = \sigma_E = 0.15$ (black/full line), $\sigma_{N_\mu} = \sigma_E = 0.1$ (red/dashed line) and $\sigma_{N_\mu} = 0.25$, $\sigma_E = 0.15$ (green/dotted line). (see details in text)

6. Conclusions

In this work we have shown that the energy evolution of shower observables are essential for the understanding of our data. The analysis of absolute values at fixed energy are very much affected by the energy scale uncertainty ($\sim 20\%$) and by the uncertainty on hadronic interaction models. This complicates considerably any interpretation in terms of mass composition. Moreover, these two uncertainties make the efficiency for selecting pure mass composition samples very low.

Through the analyses of the energy evolution of shower observables, we become rather insensitive to the absolute energy scale and the current hadronic interaction models. Therefore, we have conceived a method that is quantitatively sensitive to mass composition transitions. With this method we can validate/refute mass composition scenarios that are, for instance, able to explain the $X_{\text{max}}$ data, by searching for the corresponding transition in the number of muons measured at ground.

In fact, we have tested this method – designated as Landscape Plots – in the mass composition scenario suggested by our $X_{\text{max}}$ data (should one believe in SIBYLL2.1). The results obtained show that $RMS(N_\mu)$ is by far the best variable to look for transitions in such a scenario, while the $\langle N_\mu \rangle$ present a very low sensitivity.

The method developed in this note was used to provide requirements on the event-by-event resolution of the number of muons at ground and on the energy determination. We found that if
one wants to obtain information from the $\langle N_\mu \rangle$ (i.e. reach at least 3$\sigma$, with 100 events at $E = 10^{19}$ eV), the $\sigma_{N_\mu}$ should be push down to 10%, while the energy resolution should be a few percent bellow 15%. Moreover, we found that the mass composition scenario suggested by the $X_{\text{max}}$ data, when interpreted with Sibyll2.1, should present a transition on the $\text{RMS}(N_\mu)$ at $E \sim 10^{19}$ eV. This transition should be observed with a significance of more than 5$\sigma$ for $\sigma_{N_\mu} \sim \sigma_E \sim 0.15$ and with the equivalent of roughly one year of operation of the full SD-array$^9$.

Finally, it should be noted that while the obtained results give an indication of the necessary resolutions to assess mass composition scenarios, the precise value of the transition significances requires a full simulation which includes all the detector effects. There are some effects that can probably still be included in this simple approach to give a better estimation of the transition significance, such as the impact of energy bins migration, and these will be the subject of future updates to this work.

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References


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$^9$without considering quality cuts.
Appendixes

In these appendixes we show all the relevant plots for the assessment of the three distinct mass composition scenarios that were analysed in this paper, namely: the proton to iron abrupt transition at $E = 10^{19}$ eV (Appendix A), the linear transition from pure proton to pure helium (Appendix B), and the mass composition scenario suggested by the $X_{\text{max}}$ data when interpreted with Sibyll2.1 (Appendix C). The results on the two first moments of the $X_{\text{max}}$ and the number of muons at ground distributions are displayed for each scenario.

As an additional remark, we would like to note that the scales of the 2D-plot of $D_{\text{slope}}$ (the Landscape Plot for the transition significances) were fixed to $D_{\text{slope}} \in [-10\sigma; 10\sigma]$. In this way one can do a direct comparison between the different observables. In some particular cases the value in the bin exceeds the this range and the bin will appear as white. The value of these bins can be roughly recovered through the analyses of the corresponding profiles exhibited in the plot at the right of the 2D-plot.

Appendix A. Proton to Iron abrupt transition

![Figure A.17](image-url)
Figure A.18:

Figure A.19:
Figure A.20:

Figure A.21:
Figure A.22:
Appendix B. Proton to Helium transition

Figure B.23:

Figure B.24:
Figure B.25:

Figure B.26:
Figure B.27:

Figure B.28:
Figure B.29:

Figure B.30:
Appendix C. $X_{\text{max}}$ data suggested scenario

Figure C.31:

Figure C.32:
Figure C.33:

Figure C.34:
Figure C.35:

Figure C.36:
Figure C.37:

Figure C.38: t